Furriness and Funds Distribution in Portfolio Optimization

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INTRODUCTION

In this paper, I designed a furry system so that customers are classified to belong to any one of the following three categories: 1

***Conservative and security-oriented (risk shy)**

***Growth-oriented and dynamic (risk neutral)**

***Chance-oriented and progressive (risk happy).**

Besides being useful for customers, investor categorization is useful for the proficient investment professionals as well. Most brokerage houses would consider this information as significant as it gives them a way of targeting customers with a variety of financial products more successfully - including insurance, saving schemes, mutual funds, and so forth. Overall, countless conscientious brokerage houses understand that if they provide an efficient service that is tailored to individual wants, in the long-term there is distant possibility that they will hold no substance for their customers whether the market is up or down. Yet, even if it may be true that investors can be classified according to a limited number of categories based on theories of personality already in the psychological career's munitions store, it must be said that these categorization schemes based on the Behavioral Sciences are still very much in their immaturity and they may still endure from the dilemma of their significances being analogous to other related typographies, as well as of greatly oversimplifying the different investor behaviours.

EXPLORING THE INFERENCES OF UTILITY THEORY ON INVESTOR CATEGORIZATION

In my current effort, I have used the proverbial gibbet of neo-classical utility theory to seek and formulate a composition scheme for investor categorization according to the effectiveness favorites of individual investors (and also possible re-ordering of such preferences). The theory of customer behavior in contemporary microeconomics is exclusively established on noticeable effectiveness favorites, eliminating self-indulgent and meditative portions of utility. According to contemporary utility theory, utility is a depiction of a set of mutually unswerving preferences and not a clarification of an alternative. The fundamental loom is to ask an individual to divulge his or her individual utility predilection and not to extract any numerical appraise. [1] However, the protrusions of the significances of the opportunity that we face and the succeeding alternatives that we make are outlined by our memories of past incidents - that "the mind's eye sees the outlook from the beginning to the end; the light sorted by the past". However, this reminiscence frequently leans to be moderately selective. [9] An investor who distributes a bulky fraction of his or her funds to the risky asset in phase t-1 and makes a momentous gain will perhaps be stimulated to put an even larger fraction of the accessible funds in the risky asset in phase t. So this depositor may be said to have presented a very pathetic risk-loathing approach upto stage t, his or her acts being essentially indomitable by precedent happenings one-period back. There are two elucidations of utility - normative and optimistic. *Normative utility* challenges that *most favorable* decisions do not for all time replicate the *best* conclusions, as maximization of instantaneous utility based on discriminatory reminiscence may not inevitably involve maximization of total utility. This is exact in countless cases, especially in the vicinity of health economics and social preference theory. However, while I will be pertaining to utility theory to the very precise vicinity of funds allotment between risky and risk-less investments (and investor categorization based on such portion), we will be apprehensive with *optimistic utility,* which judges the best possible decisions as they are, and not as what they should be. I simply concentrated in using utility functions to categorize a person investor's approach towards comportment risk at a prearranged point of time. Given that the neo-classical utility preference approach is an important one, we feel it is absolutely more acquiescent to

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recognized investigation for our purpose as compared to the philosophical conceptualizations of uncontaminated licentiousness if we can accept decision utility preferences engendered by discriminatory remembrance. If f is a specified utility function and c is the wealth coefficient, then we have E [f (c+ r)] = f [c + E (r) - d], that is, E [f (c + r)] = f(c - d), where r is the outcome of a risky undertaking given by a known probability distribution whose expected value E (r) is zero. Since the conclusion of the risky undertaking is as probable to be positive as negative, we would be agreeable to recompense a small amount d, the *risk premium,* to avoid having to undertake the risky venture. Intensifying the utilities in *Taylor series* to second categorize on the left-hand side and to first categorize on the right-hand side and subsequent algebraic simplification leads to the universal formula d = - (v/2) f''(c)/f' (c), where v = C (r2) is the variance of the feasible outcomes. This demonstrates that fairly accurate risk premium is comparative to the variance - an impression that carries a parallel connotation in the mean-variance theorem of classical portfolio theory. The quantity - f''(c)/f'(c) is named the *absolute risk aversion*. [6] The nature of this absolute risk aversion depends on the appearance of a definite utility function. For occasion, for a logarithmic utility function, the absolute risk aversion is reliant on the wealth coefficient c, such that it decreases with an increase in c. On the other hand, for an exponential utility function, the absolute risk aversion becomes a constant equal to the reciprocal of the risk premium.

THE NEO-CLASSICAL UTILITY MAXIMIZATION APPROACH

In its simplest form, it may be properly characterizing an individual investor's utility maximization objective as the following mathematical programming problem:

Maximize U = f (x, y)

Subject to x + y = 1,

x e **0 and y is unrestricted in sign**

Here x and y position for the fractions of invest gifted funds distributed by the investor to the market portfolio and a risk-free securities. The last restriction is to guarantee that the investor cannot at all scrounge at the market rate to invest in the risk-free asset, as this is evidently idealistic - the market rate being evidently higher than the risk-free rate. However, *a blatantly antagonistic investor can borrow at the risk-free rate* to invest in the market portfolio. In investment vernacular, this is acknowledged as ***leverage.*** [5] As in classical microeconomics, we may solve the above predicament using the *Lagrangian multiplier* technique. The transformed Lagrangian function is as follows:

Z = f (x, y) + ⌊ (1-x-y) (1)

By the first order (compulsory) circumstance of maximization, we originate the following scheme of linear algebraic equations:

Zx = fx - ⌊ = 0 …(i)

Zy = fy - ⌊ = 0 …(ii)

Z⌊ = 1 - x - y = 0 …(iii) (2)

The investor's symmetry is then acquired as the circumstance **fx = fy =** ⌊ *. ⌊ * may be conservatively construed as the *marginal utility of money* (i.e. the investable funds at the clearance of the individual investor) when the investor's utility is maximized. [2 the individual investor's unresponsiveness curvature will be obtained as the locus of all arrangements of x and y that will yield an unvarying level of utility. Mathematically stated, this basically simmers down to the following total differential:

dU = fxdx +fydy = 0 (3)

The instantaneous connotation of (3) is that dy/dx = -fx/fy, i.e. assuming (fx, fy) > 0; this gives the negative angle of the individual investor's indifference curve and may be consistently interpreted as the *marginal rate* *of replacement of allocable funds* between the market portfolio and the risk-free asset. A second order (sufficient) circumstance for maximization of investor utility may be also resultant on a similar procession as that in economic theory of consumer behavior, by means of the sign of the permeated Hessian determinant, which is given as follows:

—

H| = 2®x®yfxy - ®y

2fxx - ®x

2fyy (4)

In the above equation, ®x and ®y stand for the coefficients of x and y in the constraint Equation. In this case we have ®x = ®y = 1. Equation (4) therefore reduces to:

—

|H| = 2fxy - fxx - fyy (5)

—

If |H| > 0 then the stationary value of the utility function U* will be said to have attained its maximum. To demonstrate the relevance of classical utility theory in investor classification, let the utility occupation of a coherent investor be characterized by the following utility function:

U (x, y) = ax2 - by2; where

x = fraction of funds invested in the market portfolio; and

y = fraction of funds invested in the risk-free asset.

Quite evidently, x + y = 1 since the proficient portfolio must consist of an arrangement of the market portfolio with the risk-free asset. The dilemma of funds allotment within the efficient portfolio then happen to that of maximizing the prearranged utility function matter to the efficient portfolio constriction.

J. Tobin's mentioned his views through ***Separation Theorem;*** that is, investment is a two-segments progression with the dilemma of portfolio assortment which is deemed autonomous of an individual investor's utility inclinations (i.e. the first phase) to be treated *independently* from the problem of funds allotment within the selected portfolio which is reliant on the individual investor's utility function (i.e. the second phase). Using this concept, we can mathematically categorize all individual investor attitudes towards bearing risk into any one of three distinct classes:

Class A+: "Overtly Aggressive"

Class A: "Aggressive"

Class B: "Neutral"

Class C: "Conservative"

The dilemma is then to discover the universal point of maximum investor utility and consequently originate a mathematical basis to pigeonhole the investors into one of the three classes depending upon the most favorable values of x and y. The original problem can be stated as a classical non-linear programming with a single egalitaria nism restriction as follows:

Maximize U (x, y) = ax2 - by2

Theme to:

x + y = 1,

x e **0 and y is unhindered in indication**

We set up the following malformed Lagrangian intention function:

Maximize Z = ax2 - by2 + ⌊ (1 - x - y)

Theme to:

x + y = 1,

x e **0 and y is unobstructed in sign, (where ⌊ is the Lagrangian multiplier)**

By the accustomed first-order (necessary) situation we therefore get the following arrangement of linear algebraic equations:

Zx = 2ax - ⌊ = 0 ... (i),
Zy = -2by - ⌊ = 0 ... (ii); and
Z⌊ = 1 - x - y = 0 ... (iii) (6)

Solving the above arrangement we get x/y = -b/a. But x + y = 1 as per the funds restriction.

Therefore (-b/a) y + y = 1 i.e. **y* = [1 + (-b/a)]-1 = [(a-b)/a]-1 = a/(a-b).**

Now substituting

for y in the restriction equation, we get **x* = 1-a/(a-b) = -b/(a-b).** Therefore the motionless value of the utility function is **U* = a [-b/(a-b)] 2 - b [a/(a-b)] 2 = -ab/(a - b).**

Now, fxx = 2a, fxy = fyx = 0 and fyy = -2b. Therefore, by the second order (sufficient) circumstance, we have:

—

|H| = 2fxy - fxx - fyy = 0 -2a - (-2b) = 2 (b - a) (7)

Therefore, the bordered Hessian determinant will be positive in this case if and only if we have (a - b) < 0. That is, given that a < b, our chosen utility function will be maximized at U* = ax*2 - by*2. However, the satisfaction of the non-negativity constraint on x*

would require that b > 0 so that - b < 0; thus yielding [- b / (a - b)] > 0.

Classification of investors:

Class Basis of determination
A+ (y*< x*) and (y* d **0)**
A (y*< x*) and (y*> 0)
B (y*= x*)
C (y*> x*)

(I.3)Outcome of a risk-free asset on investor utility

The leeway to loan or scrounge money at a risk-free rate broadens the assortment of investment options for an individual investor. The insertion of the risk-free asset makes it probable for the investor to select a portfolio that *dictates* any other portfolio made up of only risky securities. This implies that an entity investor will be able to accomplish a higher *indifference curve* than would be potential in the absence of the risk-free asset. The risk free asset makes it potential to *split* the investor's decision-making progression into two divergent segments - identifying the market portfolio and funds allocation. The market portfolio is the portfolio of risky assets that includes each and every available risky security. As all investors who clasp any risky assets at all will choose to clutch the market portfolio, this alternative is self-sufficient of an individual investor's utility predilections. Now, the expected return on a two-security portfolio connecting a risk-free asset and the market portfolio is given by E (Rp) = xE (Rm) + yRf; where E (Rp) is the expected return on the optimal portfolio, E (Rm) = expected return on the market portfolio; and Rf is the return on the risk-free asset. Obviously, x + y = 1. Substituting for x and y with x* and y* from our illustrative case, we therefore get:

E (Rp)* = [-b/(a-b)] E (Rm) + [a/(a-b)] Rf

(8) As may be confirmed spontaneously, if b = 0 then of itinerary we have E (Rp) = Rf,
as in that case the finest value of the effectiveness function too is concentrated to
U* = -a0/(a-0) = 0.

The equation of the Capital Market Line in the innovative description of the CAPM may be remembered as **E (Rp) = Rf + [E (Rm) - Rf](Sp/Sm);** where E (Rp) is expected return on the efficient portfolio, E (Rm) is the expected return on the market portfolio, Rf is the return on the risk-free asset, Sm is the standard deviation of the market portfolio returns; and Sp is the standard deviation of the efficient portfolio returns. Now, equating for E (Rp) with E (Rp)* we therefore get:

Rf + [E (Rm) - Rf](Sp/Sm) = [-b/(a-b)] E (Rm) + [a/(a-b)] Rf, i.e.

Sp* = Sm [Rf {a/(a-b) - 1} + {-b/(a-b)} E (Rm)] / [E (Rm) - Rf]

= Sm [E (Rm) - Rf][-b/(a-b)] / [E (Rm) - Rf]

= Sm [-b/(a - b)] (9)

This mathematically exhibits that a balanced patron having a quadratic utility function of the form U = ax2 - by2, at his or her point of maximum utility (i.e. affinity to return combined with averseness to risk), presumes a given efficient portfolio risk (standard deviation of returns) correspondent to Sp* = Sm [-b/(a - b)]; when the efficient portfolio consists of the market portfolio coupled with a risk-free asset. The investor in this case, will be classified within a fastidious category A, B or C according to whether -b/(a-b) is greater than, equal in value or lesser than a/(a-b), given that a < b and b > 0.

Case I: (b > a, b > 0 and a >0)

Let b = 3 and a = 2. Thus, we have (b > a) and (-b < a). Then we have x* = -3/(2-3) = 3 and y* = 2/(2-3) = -2. Therefore (x*>y*) and (y*<0). So the investor can be classified as Class A+.

Case II: (b > a, b > 0, a < 0 and b > |a|)

Let b = 3 and a = - 2. Thus, we have (b > a) and (- b < a). Then, x* = -3/(-2-3) = 0.60 and y* = -2/(-2-3) = 0.40. Therefore (x* > y*) and (y*>0). So the investor can be re-classified as Class A!

Case III: (b > a, b > 0, a < 0 and b = |a|)

Let b = 3 and a = -3. Thus, we have (b > a) and (b = |a|). Then we have x* = -3/(-3-3) = 0.5 and y* = -3/(-3-3) = 0.5. Therefore we have (x* = y*). So now the investor can be reclassified as Class B!

Case IV: (b > a, b > 0, a < 0 and b < |a|)

Let b = 3 and a = -5. Thus, we have (b>a) and (b<|a|). Then we have x* = -3/(-5-3) = 0.375 and y* = -5/(-5-3) = 0.625. Therefore we have (x* < y*). So, now the investor can be re-classified as Class C! So we may see that even for this relatively simple utility function, the vital categorization of the investor enduringly into any one risk-class would be impracticable as the assortment of values for the coefficients a and b could be buttoning energetically from one variety to another as the investor tries to fiddle with and re-adjust his or her risk-demeanor attitude. This makes the neo-classical loom unsatisfactory in itself to disembark at a categorization. Here arranges the rationalization to bring in a flattering, ***fuzzy modeling*** loom. Moreover, if we bring in time itself as an independent variable into the utility maximization skeleton, then one preference variable (weighted in favor of risk-avoidance) could be viewed as a *controlling factor* on the other choice variable (weighted in favor of risk acceptance). Then the resulting problem could be gainfully explored in the light of ***optimal control theory.***

(II.1) Modeling fuzziness in the funds allocation behavior of an individual investor.

The frontier between the favorite sets of an individual investor, for funds distribution between a risk-free asset and the risky market portfolio, tends to be rather furry as the investor continually evaluates and shifts his or her position; unless it is a passive ***buy and - hold*** kind of portfolio. Thus, if the cosmos of communication is **U = {C, B, A and A+}** where C, B, A and A+ are our four risk classes "conservative", "neutral", "aggressive" and "overtly aggressive" correspondingly, then the furry subset of U given by **P = {x1/C, x2/B, x3/A, x4/A+}** is the *true* penchant set for our intentions; where we have **0** d **(x1, x2, x3, x4)** d **1,** all the symbols having their usual meanings. Although theoretically any of the P (xi) values could be equal to harmony, in authenticity it is far more likely that P (xi) < 1 for i = 1, 2, 3, 4 i.e. the fuzzy subset P is most likely to be *subnormal*. Also, similarly, in most real-life cases it is expected that P (xi) > 0 for i = 1, 2, 3, 4 i.e. all the elements of P will be included in its *support*:

supp (P) = {C, B, A, A+} = U.

The decisive position of investigation is absolutely the individual investors preference ordering i.e. whether an investor is ***principally conservative or mostly aggressive.*** It is understandable that a mainly conservative investor could behave aggressively at times and vice versa but in common their behavior will be in stroke with their categorization. So the categorization often depends on the height of the furry subset

P: **height (P) = MaxxP (x).**

So one would think that the risk-neutral class becomes largely redundant, as investors in general will lean to catch classified as either mainly conformist or principally aggressive. However, as already said, in reality, the element B will also generally have a *non-zero degree of membership* in the furry rift and hence cannot be dropped.

The fuzziness adjoining investor classification curtails from the fuzziness in the predilection relatives concerning the allotment of funds between the risk-free and the risky assets in the optimal portfolio. It may be mathematically explained as follows:

Let **M** be the set of allocation options open to the investor. Then, the fuzzy preference relation is a fuzzy subset of the **M x M** space identifiable by the following membership function:

⎛**R (mi, mj) = 1;** mi is definitely preferred to mj

c• (0.5, 1); mi is somewhat preferred to mj

0.5; point of perfect neutrality

d • (1, 0.5); mj is somewhat preferred to mi; and

0; mj is definitely preferred to mi (10)

The fuzzy predilection relation is assumed to meet the essential circumstances of reciprocity and transitivity. However, owing to extensive puzzlement regarding tolerable working definition of transitivity in a fuzzy set-up, it is often completely neglected thereby leaving only the reciprocity assets. These assets may be succinctly represented as follows:

⎛**R (mi, mj) = 1 -** ⎛**R (mj, mi), •i ¯j** (11)

If we are to further assume a sensible cardinality of the set **M,** then the predilection relation Rv of an individual investor v may also be written in a matrix form as follows:

[12]

[rijv] = [⎛**R (mi, mj)], •i, j, v (12)**

Typically, given the proficient frontier and the risk-free asset, there can be one and only one optimal portfolio analogous to the point of tangency between the risk-free rate and the rounded efficient frontier. Then fuzzy sense modeling framework does not in any way agitates this fragment of the classical skeleton. The fuzzy modeling, like the classical Lagrangian multiplier method, comes in only *after* the optimal portfolio has been recognized and the dilemma facing the investor is that of allocating the obtainable funds between the risky and the risk-free assets subject to a governing budget restriction. The investor is theoretically faced with a countless number of potential arrangements of the risk-free asset and the market portfolio but the eventual allotment depends on the investor's utility function to which we now extend the fuzzy preference relation. The available choices to the investor given his or her utility preferences decide the creation of discourse. The more hesitant are the investor's utility preferences, the wider is the range of available choices and the greater is the degree fuzziness involved in the preference relation, which would then extend to the investor classification. Also, wider the range of available choices to the investor, the higher is the expected information content or *entropy* of the allocation decision.

(II.2) Entropy As a Appraise of Furriness

The word entropy takes places in resemblance with *thermodynamics* where the essential idiom has the following mathematical form:

S = k log b ⌉ (13)

In thermodynamics, entropy is connected to the *extent of disarray* or arrangement possibility⌉ of the canonical gathering. Its use engages an investigation of the microstates' allocation in the canonical congregation among the offered energy levels for both isothermal reversible and isothermal irreversible practices (with a presence modification). The physical balance factor k is the *Boltzmann constant*. [7] However, the thermodynamic outline has a diverse symbol and the expression *negentropy* is therefore sometimes used to designate projected information. Though Claude Shannon initially conceptualized the entropy measure of expected information, it was DeLuca and Termini who brought this concept in the dominions of furry mathematics when they sought to obtain a universal mathematical measure of fuzziness.

Let us consider the fuzzy subset F = {r1/X, r2/Y}, 0 δ (r1, r2) δ 1, where X is the event (y<x) and Y is the event (yεx), x being the fraction of funds to be invested in the market portfolio and y being the section of funds to be invested in the risk-less security. Then the

DeLuca-Termini conditions for evaluation of fuzziness may be stated as follows: [3]

FUZ (F) = 0 if F is a crunchy set i.e. if the investor confidential under a particular risk group *always* invests intact funds either in the risk-free asset (conservative attitude) or in the market portfolio (aggressive attitude)

FUZ (F) = Max FUZ (F) when F = (0.5/X, 0.5/Y)

FUZ (F) e **FUZ (F*)** if F* is a *sharpened version* of F, i.e. if F* is a fuzzy detachment satisfying F*(ri) ε F (ri) given that F (ri) ε 0.5 and F (ri) ε F*(ri) given that 0.5 ε F (ri). The second circumstance is straight derived from the idea of entropy. Shannon's measure of entropy for an n - events case is given as follows: [10]

H = - k © (pi log pi), where we have ©pi = 1 (14)

The Lagrangian form of the above function is as follows:

HL = - k ©(pi log pi) + ⌊ **(1 - ©pi)** (15)

Taking partial derivatives w.r.t. pi and setting equal to zero as per the necessary condition of maximization, we have the following stationary condition:

HL = -k [log pi +1] - ⌊ **= 0** (16)

pi

It may be derived from (16) that at the point of maximum entropy, log pi = -[(⌊/k)+1], i.e. log pi becomes a constant. This means that at the point of maximum entropy, pi becomes independent of the i and equalized to a constant value for i = 1, 2, ..., n. In an n-events case therefore, at the point of maximum entropy we necessarily have:

p1 = p2 = … = pi = … = pn = 1/n (17)

For n = 2 therefore, we obviously have the necessary condition for entropy maximization as p1 = p2 = ½ = 0.5. In terms of the Fuzzy preference relation, this boils down to exactly the second DeLuca-Termini condition. Keeping this close relation with mathematical information theory in mind, DeLuca and Termini even went on to incorporate Shannon's entropy measure as their chosen measure of fuzziness. For our portfolio funds allocation model, this measure could simply be stated as follows:

FUZ (F) = - k [{F(r1) log F(ri) + (1-F(r1)) log (1-F (r1))}+ {F(r2) log F(r2) + (1-F(r2)) log (1-F(r2))}] (18)

(II.3) Metric Measures of fuzziness.

Perhaps the best technique of measuring furriness will be through the dimension of the detachment between F and Fc, as furriness is scientifically equivalent to the lack of dissimilarity between a set and its complement. In terms of our portfolio funds distribution model, this is equivalent to the ambivalence in the mind of the individual investor concerning whether to put a larger or smaller fraction of offered funds in the risk-less asset.The higher this ambivalence, the closer F is to Fc and greater is the fuzziness. This measure may be assembled for our case by considering the furry subset F as a vector *with 2 components*. That is, F (ri) is the ith component of a vector in lieu of the furry subset F and (1 - F (ri)) is the it constituent of a vector representing the corresponding fuzzy subset Fc. Thus letting D be a metric in 2 space; we have the detachment between F and Fc as follows: [11]

Dλ (F, Fc) = [© |F (ri) - Fc (ri)|λ] 1/λ, where λ = 1, 2, 3, … (19)

For Euclidean Space with λ =2, this metric becomes very similar to the statistical variance measure RMSD (root mean square deviation). Moreover, as Fc (ri) = 1 - F (ri), the above formula may be written in a simplified approach as follows:

Dλ (F, Fc) = [© |2F (ri) - 1|λ] 1/λ, where λ = 1, 2, 3, … (20)

For λ =1, this becomes the *Hamming metric* having the following form:

D1 (F, Fc) = © |2F(ri) - 1| (21)

If the investor forever puts a greater percentage of funds in either the risk-free asset or the market portfolio, then F is abridged to a crusty set and |2F (ri) - 1| = 1. Based on the above metrics, a collective appraise of furriness may now be defined as follows for our portfolio funds allocation model. This is done as follows: For a crunchy set F, Fc is truly corresponding, meaning that the metric detachment becomes: **Dλ *(F, Fc) = 21/λ,** where λ = 1, 2, 3, … (22)

An effective measure of fuzziness could therefore be as follows:

FUZλ (F) = [21/λ - Dλ (F, Fc)]/ 21/λ = 1 - Dλ (F, Fc)]/ 21/λ (23)

For the Euclidean metric we would then have:

FUZ2 (F) = 1 - [© (2F(ri) - 1) 2] ½

→2 = 1 - (→ 2)(RMSD), where **RMSD = ([© (2F(ri) - 1) 2] ½)/2** (24)

For the Hamming metric, the formula will simply be as follows:

FUZ1 (F) = 1 - © |2F(ri) - 1| 2 (25)

Having worked on the relevant appraise for the degree of furriness of our leading preference relative, we dedicate the next sector of our present paper to the incorporation of neurofuzzy control systems to fabricate and work out risk classification for the homological utility of an investor under different circumstances. In a succeeding section, we also take a passing look at the possible request of optimal control theory to model the vibrant of funds allotment behavior of an individual investor.

(IV) Exploring time-dependent funds allocation behavior of individual investor in the light of optimal control theory.

If the inter-temporal utility of an individual out looked from time t is recursively defined as

Ut = W [ct, æ (Ut+1| It)], then the aggregator function W makes recent inter-temporal utility a function of recent expenditure ct and of a conviction correspondent of next period's random utility. If that is computed using information up to t. Then, the individual could prefer a control variable xt in period t to maximize Ut. [4] In the circumstance of the mean variance model, a appropriate candidate for the control variable could well be the fraction of funds set aside for investment in the risk-free asset. So, the objective function would incorporate the investor's total temporal utility in a given time range [0, T]. Given that we include time as a continuous variable in the model, we may successfully formulate the problem applying classical optimal control theory. The conceivable methodology for formulating this model is what we shall investigate in this section.

The basic most favorable control dilemma can be stated as follows: [8] Find the control vector **u** = (u1, u2, …, um) which optimizes the functional, called the *performance index*, J = + f0 (**x, u,** t) dt over the range (0, T), where **x** = (x1, x2, …, xn) is called the state vector, t is the time parameter, T is the terminal time and f0 is a specified function of **x, u** and t. The state variables xi and the control variables ui are related as dxi/dt = fi (x1, x2, …, xn; u1, u2, …, um; t), i = 1, 2, …, n. In many control problems, the arrangement is linearly expressible as **x (.)** = [A] nxn **x** + [B] nxm **u**, where all the symbols have their usual implications. As an illustrative example, we may again consider the quadratic function that we used earlier f0 (x, y) = ax2 - by2. Then the problem is to find the control vector that makes the performance index J = +(ax2 - by2) dt stationary with x = 1 - y in the range (0, T). The ***Hamiltonian*** may be expressed as ***H*** = **f0** + ⌊**y** = **(ax2 - by2)** + ⌊**y**. The standard resolution technique yields -*H*x = ⌊ (.) … (i) and *H*u = 0 … (ii) whereby we have the following system of equations: -2ax =⌊ (.) … (iii) and -2y + ⌊ = 0 … (iv). Differentiation of (iv) leads to -2y(.) + ⌊ (.) = 0 … (v). Solving (iii) and (v) simultaneously, we get 2ax = -2y(.) = -⌊ (.) i.e. y(.) = -ax … (vi). Transforming (iii) in terms of x and solving the resulting ordinary differential equation would yield the state route x (t) and the optimal control u (t) for the specified quadratic utility function, which can be easily done by most standard mathematical computing software packages. So, given a fastidious form of a utility function, we can outline the energetic time-path of an individual investor's fund portion behavior (and hence; his or her classification) within the domain of the mean-variance model by acquiring the state course of x - the proportion of funds invested in the market portfolio and the equivalent control variable y - the fraction of funds invested in the risk-free asset using the standard observes of finest control theory.

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