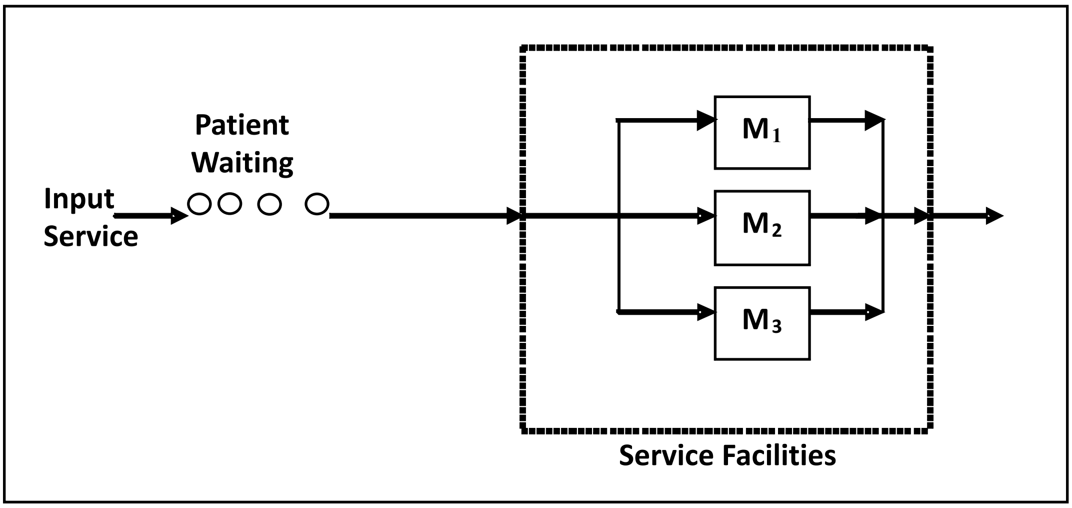
Hospital Central Laboratory's Manpower Planning By Use Of Queueing Theory

*Prasanta Kumar Brahma

THE PROBLEM

The function of the technicians in a hospital central laboratory is to collect the samples and to investigate these samples under the supervision of specialist doctors and prepare the report. When ever a patient arrives, the samples are collected by the technician of the sample collection counter and after that, he sends those samples to different departments of the central laboratory such as Pathology, Biochemistry and Microbiology etc.. After receiving the samples the technician of the concerned departments process those samples under the supervision of the specialist doctors and prepares the reports. The service facility is arranged in parallel; as in Fig.-1: technicians M1, M2, M3 etc. may be facilities, and incoming patients are served by any free technician. If no technician is free, patients must wait and may thus accumulate, forming a queue. Patients are usually all given the same priority, so that service is on a first come first served basis.

Fig-1: The Queueing System: One Line Multiple Server In Parallel



In addition to these routine services, the unit provides non-routine services scheduled at definite times. It is not a simple task to estimate man hours to be allocated to non-routine activities: patients arrive at random, and the time required to complete the service is also random, depending on the type of test. Hiring an unlimited number of technicians would allow immediate service for patients at any time but is obviously not economically sound. On the other hand, significant under staffing of the unit would result in poor service. The choice of staffing level must be consistent with acceptable labor cost and with acceptable waiting time for service of patients.

DATA COLLECTION AND ANALYSIS

To apply queueing theory to the problem, it is necessary to know the distribution of the arrival time and service time and queue discipline (in this case first come first served, as indicated). To determine arrival time and service time, distribution data were collected from 8AM to 3PM-the busiest part of the day over a two weeks period: the time was noted as each request for service arrived; and also when the technician started investigation and completed the report preparation with the Doctor's signature. The difference between the latter two time was defined as service time.
Data were obtained for 375 patients for service; these were grouped by time required to perform the service, as shown

*Lecturer cum Jr. MRO, IMS & SUM Hospital, Bhubaneswar. Email : brahma_pk@rediffmail.com

in Table-1. The mean service time was calculated as 11.9 min.; for convenience, 12 min. was taken, as an approximation. The service time is compared with an exponential distribution calculated from

$$T(>t) = N e^{-t/m}$$

where t is the time in minutes , N is the number of patients observed (375), and m is the mean service time. A goodness of fit test gave a χ^2 value of 12.4, while the critical value of χ^2 (α=0.05, d.f. 7) is 14.07, indicating that the observed service times conform to the theoretical exponential distribution. The data were then tabulated by number of patients arriving during each consecutive 12 min. Intervals (i.e. per mean service time interval) was determined to be 10.7. On the assumption of a Poisson arrival process, the theoretical frequency distribution of arrival per men service time was calculated as :

$$T(x) = \frac{10.7 \, M \, e^{-10.7}}{x!}$$

where x = patients arrived per mean service times and M = 315, the number of service time intervals observed. The observed and the calculated frequency distributions are shown in Table-2 ; a goodness of fit test of these distributions gave a χ^2 value of 1.7, while the critical value of χ^2 (α=0.05, d.f. 5) is 11.07, and justifying the assumption of a Poisson process for the arrival of patients.

APPLICATION OF THE QUEUEING MODEL

It has been established that the technician's service processes the characteristics of a Poisson arrival process, an exponential distribution of service times, a first come first served queue discipline, and a service arrangement of multiple servers acting in parallel. It is assumed that the system operators are in a steady state condition i.e. that the current state is independent of the initial conditions. Under these circumstances, it may be assumed that a simple queueing model will describe the service adequately. No attempt is made here to develop the mathematical model in detail. The interested reader will find detailed discussions of the formulas in the works of Hillier and Lieberman, Mase and Saaty.

STAFFING LEVEL AND SERVICE DELAY

For the present problem, the parts of the model that are of interest are those which express, as functions of the number of technicians, L_q, the expected queue length and P(>t), the probability that a patient will have to wait some time longer than t. Let λ denote the mean number of arrival per unit time, μ the mean number of patients investigations done by the technicians per unit time, and s the number of technicians. If the mean arrival rate is less than the service rate, $\lambda < \mu s$; for convenience, let $\rho = \lambda/\mu s$ and note that $\rho<1$. The probability that no patients are waiting can be defined as

$$P_o = \frac{1}{\left[\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!\,(1-\rho)}\right]}$$

where n is the number of patients in the queueing system. Then the waiting time probability may be calculated as

$$P(>t) = \frac{P_o (\lambda/\mu)^s}{s!\,(1-\rho)} \quad e^{-s\,\mu(1-\rho)} \quad \text{------------------------------------(1)}$$

and the expected queue length may be obtained from

$$L_q = \frac{P_o (\lambda/\mu)^s \rho}{s!\,(1-\rho)^2} \quad \text{----------------------------------(2)}$$

Expected queue length L_q was calculated from Eq.2 for various staffing levels; the results are given in Table-3.

STAFFING LEVEL AND COST

The cost of waiting is intangible in most cases and is difficult to estimate. One can, however, get an insight into the problem by arbitrarily varying the ratio of cost of waiting per unit time to cost of service per unit time and studying its effect on the system. To construct an example, let unit time be one hour, C_1 is service cost per technician per hour and C_2 is waiting cost per patient per hour.At any time [s - (λ/μ)] technicians are idle : the cost of idleness is C_1[s - (λ/μ)]. Also, L_q calls are waiting to be served with an additional cost of C_2L_q , so that the equation describing the total hourly cost is

$$F(s) = C_1 [s - (\lambda/\mu)] + C_2 L_q$$

The objective is to minimise F, which is a function of s. The equation can be solved by trial and error ; let C_1 =₹ 20/hr. , and assign alternative values 5 and 15 to the ratio C_2/C_1. The resulting values for hourly cost are shown in Table-4.

DISCUSSION

The administrator must decide which one of several criteria is the most relevant as the basis for choosing a staffing level. He may decide that a given percentage of calls may not be allowed to wait for service longer than a certain time or that no more than a certain percentage of patients may be waiting in the queue for service at any time, or he may choose to minimize the total cost of the operation, balancing the cost of waiting against the cost of providing additional technicians. With twenty technicians, 20 percent of patients would wait longer than 10 minutes, while with thirty technicians, only 2 percent of the calls would wait longer than 10 minutes. The choice of criteria must dictate which of these values is to be regarded as optimal. By inspecting Table-3, one can also decide on a tolerance limit for patients waiting to be serviced at any time. It is obvious from the results that increasing the number of technicians from twenty to thirty has marked effect on the queue length, while a further increase to forty or more has relatively little effect. Adding an additional person is not always an effective solution. With regard to the cost values shown in Table-4, it can be seen that when waiting cost (C_2) is excessively high, it is economical to have forty technicians instead of thirty. This short of result was expected intuitively. In emergency unit, therefore, a higher level of service should be provided. Such calculations can provide estimates of staffing costs for emergency service. In this particular application, it was found that the data on service times and arrival rates were adequately represented by the theoretical distributions. In more complex cases, these distributions may not approximate the real life situation: for example, in a emergency department, there may be priorities of service dependent on the condition of each newly arrived patient. The analysis of such a situation is involved and may sometimes be impossible, so that computer simulation would be necessary to provide an objective basis for decisions on staffing. The technician service problem however proved to be simple enough that the analytic approach of queueing theory was both feasible and effective: as a consequence of this analysis, it was decided that thirty technicians would be the optimal on duty staff for the first shift. With this staffing level, about 86 percent of the requests would be satisfied immediately, while 14 percent would wait an average of four minutes to be served. After the desired staffing level was achieved, departmental records showed that the technician unit was providing adequate service and that waiting times were within the desired limits. The recommendation is that reduction of manpower resulted the saving.

Table 1: Distribution Of Calls Grouped By Service Time Required

Service Time In Minutes	Number Of Patients
0-5	92
>5-10	103
>10-15	82
>15-20	39
>20-25	26
>25-30	8
>30-35	9
>35-40	16

Table 2: Distribution Of Patients Grouped By Service Time Required

Number Of Patients	Observed	Theoretical
0	96	98
1	112	113
2	74	69
3	20	24
4	10	9
5	3	2

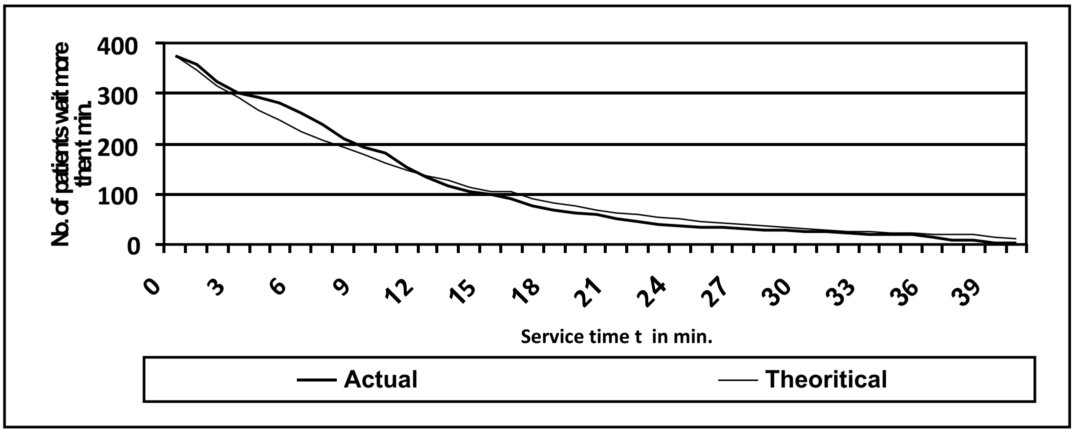
Table 3: Expected Queue Length With Various Number Of Technicians

Number Of Technicians	Calls Waiting For Service
20	0.675
30	0.094
40	0.016
50	0.003

Table 4: Hourly Cost Of Various Staffing Levels At Two Different Ratios Of Waiting Cost (C_2) To Service Cost (C_1)

Number Of Technicians	Cost Per Hour	
	C2/C1 = 5	C2/C1 = 15
20	83.5	218.2
30	45.4	64.2
40	5.76	60.8
50	7.63	76.8

Figure 1: Distribution Of Service Time



BIBLIOGRAPHY

1.Bailey, N.T.J. A study of queues and appointment systems in hospital outpatient departments with special reference to waiting time. J. Roy. Stat. Soc. (Sec.B) 14:185,1952.
2.Brockmeyer,E, H.L.Halstrom and A.Jensen. The life and works of A. K.Erlang Trans of the Danish Acad. Of Tech.Sci.2: 9, 1948.
3.Bailey, N.T.J and J.D.Welch. Appointment systems in hospital outpatient departments . Lancet 262: 1105, 1952.
4.Edie. L.C. Traffic delays at toll boths. Op.Res.4:107,1956.
5.Haussamnn, R. K. D. Waiting time as an index of quality of nursing care. Health Serv. Res. 5:92 Summer 1970.
6.Hiller.E.S. and G.J.Lieberman , Introduction to Operation Research. San Fransisco: Holden-Day.1967.
7.Morse, P. M. Queues, Inventories and Maintenance. New York: Wiley, 1958.